Appendix

A1 Proofs

Proof of Lemma 1: To prove part (i), observe that $\widehat{Q}$ maximizes $u\left(y^l - T, \overline{Q}\right) = u\left(y^l - \pi^l p\overline{Q}, \overline{Q}\right) = u\left(y^l - p\overline{Q} + \pi^h p\overline{Q}, \overline{Q}\right)$, while Q^* maximizes $u\left(y^l - p\overline{Q} + \pi^h pQ^*, \overline{Q}\right)$. Now comparison of these two expressions reveals

$$u\left(y^l - p\overline{Q} + \pi^h p\overline{Q}, \overline{Q}\right) \lesseqqgtr u\left(y^l - p\overline{Q} + \pi^h pQ^*, \overline{Q}\right) \Leftrightarrow \overline{Q} \lesseqqgtr Q^*.$$

Consequently, $u\left(y^l - p\overline{Q} + \pi^h p\overline{Q}, \overline{Q}\right)$ crosses $u\left(y^l - p\overline{Q} + \pi^h pQ^*, \overline{Q}\right)$ at $\overline{Q} = Q^*$ with a positive slope, and $\widehat{Q} > Q^*$. Second, observe that $\widetilde{Q}$ is defined as the (minimum) value of $\overline{Q}$ at which, $u(y^l - T, \overline{Q}) = v(p, y^l)$. This implies that $\widetilde{Q} < q(p, y^l)$ so that at $\widetilde{Q}$, $u_q/u_c > p$. With u_q/u_c being decreasing in q, it follows that $\widetilde{Q} < Q^*$. Finally, we have at $\overline{Q} = Q^{min}$,

$$u(y^l - T, Q^{min}) = v(p, y^l - T), \tag{A1}$$

and at $\overline{Q} = \widetilde{Q}$,

$$u(y^l - T, \widetilde{Q}) = v(p, y^l). \tag{A2}$$

Clearly, the value of the right-hand side of (A1) is less than the value of the right-hand side of (A2). Consequently, the left-hand side of (A1) will also be smaller than the left-hand side of (A2), implying that $Q^{min} < \widetilde{Q}$.

To prove part (ii), first observe that if y^h is "very close" to y^l, Q^{max} will be "very close" to Q^{min} and one may not even be able to separate the types. Under this circumstance, and given that $Q^{min} < \widetilde{Q}$, we have $Q^{max} < \widetilde{Q}$. Secondly, while Q^{max} increases with $y^h - y^l$, $\widehat{Q}$ is independent of y^h (although it does depend on the relative size of the rich to the poor in the total population). This means that even if $\overline{Q}$ is set above $\widehat{Q}$, it may still be not high enough for the rich to participate in the public provision program. Then, $Q^{max} > \widehat{Q}$.

Proof of Lemma 2: When a poor individual receives, under the Besley and Coate scheme, $\overline{Q} \lesseqqgtr Q^*$ (see Figure 2 for $\overline{Q} < Q^*$ and Figure 1 for $\overline{Q} > Q^*$), his marginal rate of substitution between quality and the numeraire at $\overline{Q}$ will be equal to $u_q/u_c \gtreqless p$. On the other hand, if instead he were to receive $p\overline{Q}$ in cash (while continuing to pay $\pi^l p\overline{Q}$ in taxes so that his net transfer is $\pi^h p\overline{Q}$), he would demand $\overline{q}(N_{BC}) = q(p, y^l + \pi^h p\overline{Q})$ such that $u_q/u_c = p$. Consequently, in going from receiving $\overline{Q}$ in kind to receiving $p\overline{Q}$ in cash and choosing his own q, the poor individual's marginal rate of substitution between quality and the numeraire decreases/remains the same/increases (i.e. his demand for q increases/remains the same/decreases) depending on $\overline{Q} \lesseqgtr Q^*$. That is, $\overline{q}(N_{BC}) = q(p, y^l + \pi^h p\overline{Q}) \gtreqless \overline{Q}$ according to $\overline{Q} \lesseqgtr Q^*$. The result on $t(N_{BC})$ then follows immediately from (6).

Proof of Proposition 1: Consider the Besley-and-Coate-equivalent conditional cash transfer policy for Q^{max} by setting $N = N_{BC}^{max} = \pi^h p Q^{max}$. The poor will be (weakly) better-off under this policy as compared to the original Besley and Coate solution. To see this, recall from Lemma 2 that resulting $\overline{q}(N_{BC}^{max}) = q(p, y^l + N_{BC}^{max})$ will be $\lesseqgtr$ than Q^{max} according to $Q^{max} \gtreqless Q^*$. If $Q^{max} > Q^*$, the *lower* $\overline{q}(N_{BC}^{max})$ offered (as compared to $\overline{Q} = Q^{max}$) makes the poor better off; if $Q^{max} < Q^*$, the *higher* $\overline{q}(N_{BC}^{max})$ offered makes the poor better off; if $Q^{max} = Q^*$, there will be no change in $\overline{q}(N_{BC}^{max})$ and the poor remain just as well-off (as compared to their position under Besley and Coate).

Next, one must check if the proposed policy change is feasible; that is, if it satisfies the incentive compatibility constraint (8) for the given values of $N = N_{BC}^{max}$, $t(N_{BC}^{max})$ and $T(N_{BC}^{max})$. To examine this, recall that Q^{max} satisfies (2), the incentive compatibility constraint for the rich under Besley and Coate, as an equality so that we have

$$u\left(y^h - T_{BC}^{max}, Q^{max}\right) = v\left(p, y^h - T_{BC}^{max}\right), \tag{A3}$$

where $T_{BC}^{max} = \pi^l p Q^{max}$. Now, with $T(N_{BC})$ under conditional cash transfers taking the same value as T_{BC}, the value of the right-hand side of (8) will be equal to that of the right-hand side of (A3). Turning to the left-hand side of (8), it will be less than the left-hand side of (A3) when $\overline{q}(N_{BC}^{max}) = q(p, y^l + N_{BC}^{max}) < Q^{max}$ (depicted in Figure 1) and more than it when $\overline{q}(N_{BC}^{max}) = q(p, y^l + N_{BC}^{max}) > Q^{max}$ (depicted in Figure 2). Consequently, the policy is feasible in the former case but not in the latter case.

Finally, observe that when $\overline{q}(N_{BC}^{max}) = q(p, y^l + N_{BC}^{max}) < Q^{max}$ and (8) is satisfied as a strict inequality, one can increase N over N_{BC}^{max} and make the poor even more well off. This proves that when $Q^{max} > Q^*$, one can always make the poor better off with a conditional cash transfer policy as compared to giving them Q^{max} under the Besley and Coate scheme. Secondly, when $Q^{max} < Q^*$so that $\overline{q}(N_{BC}^{max}) = q(p, y^l + N_{BC}^{max}) > Q^{max}$, the maximum N that one can achieve under the stipulated conditional cash transfer policy, N_{FB}^{max}, must be lower than N_{BC}^{max} in order to be incentive compatible. Now, transferring N_{FB}^{max} through conditional cash transfers instead of N_{BC}^{max} under Besley and Coate, with $N_{FB}^{max} < N_{BC}^{max}$, has two opposite implications. On the one hand, by transferring less resources to the poor, it would make them worse-off. On the other hand, by allowing the poor to purchase their most-preferred quality level from the market, it would make them better-off. The net effect is ambiguous and can go either way. An example at the end of the Appendix establishes this.

A2 Many income types

Assume there are H groups of peoples with incomes $y^1 < y^2 < \ldots < y^m < y^{m+1} < \ldots < y^H$, with the first m groups being designated as "poor" and the second $H - m$ groups as rich. By "designated" we mean the number of groups of people the government wishes to redistribute to. Let π^l denote the proportion of the l-type poor ($l = 1, 2, \ldots, m$), and π^h the proportion of the h-type rich ($h = m + 1, m + 2, \ldots, H$), in the population so that $\sum_{l=1}^{m} \pi^l + \sum_{h=m+1}^{H} \pi^h = 1$.

Providing one variety of q: Assume that only one variety of the indivisible good, coupled with one value of t, is offered to everyone. To characterize the utility possibility frontier, one determines $\overline{q}, t$ and T in order to maximize

$$\sum_{l=1}^{m} \gamma^l u(y^l - T + t, \overline{q}) + \sum_{h=m+1}^{H} \gamma^h v(p, y^h - T),$$

where γ^j's are positive constants such that $\sum_{j=1}^{H} \gamma^j = 1$, subject to the government's budget constraint

$$T - (p\overline{q} + t) \sum_{l=1}^{m} \pi^l \geq 0, \tag{A4}$$

and the "appropriate" incentive compatibility constraints for the m poor-type and the $H - m$ rich-type groups as discussed below.

Despite the existence of many groups of poor and rich people, our earlier assumption on the normality of q implies that we need only to consider two incentive compatibility constraints: that of the most wealthy poor and the one for the least wealthy rich. The following lemma establishes this point.

Lemma A1 *The single-crossing property. If the incentive compatibility constraint* $v(p, y - T) \geq u(y - T + t, \overline{q})$ *is binding for an individual with income* y*, it must be slack for all individuals with incomes greater than* y*:* $v(p, z - T) > u(z - T + t, \overline{q})$*, when* $z > y$*. Similarly, if* $u(y - T + t, \overline{q}) \geq v(p, y - T)$ *is binding for* y*, then* $u(z - T + t, \overline{q}) > v(p, z - T)$*, for* $z < y$*.*

Proof. Consider the downward incentive compatibility constraint (10),

$$\Delta \equiv v(p, y - T) - u(y - T + t, \overline{q}) \geq 0.$$

Substitute for t from the government's budget constraint (A4) into the above expression to arrive at

$$\Delta = v(p, y - T) - u\left(y + \frac{\sum \pi^h}{\sum \pi^l} T - p\overline{q}, \overline{q}\right). \tag{A5}$$

Partially differentiate (A5) with respect to y. We have,

$$\frac{\partial \Delta}{\partial y} = v_y(p, y - T) - u_c(y + T \sum \pi^h / \sum \pi^l - p\overline{q}, \overline{q}). \tag{A6}$$

Evaluate (A6) at the value for y that makes (10) binding so that $(c(p, y-T), q(p, y-T))$ and $(y+T\sum\pi^h/\sum\pi^l - p\overline{q}, \overline{q})$ are on the same indifference curve. Observe that $\overline{q}$ is less than efficient for the individual with income y so that $q(p, y-T) > \overline{q}$. To determine the sign of $v_y - u_c$ along an indifference curve as q increases, differentiate $u_c(c, q)$ with respect to q. We have,

$$\frac{\partial u_c}{\partial q} = u_{cc}\frac{\partial c}{\partial q} + u_{cq} = \frac{u_{cq}u_c - u_{cc}u_q}{u_c} > 0,$$

where the sign follows from normality of q. Consequently, $\partial\Delta/\partial y > 0$ which proves the first part of the Lemma. A similar argument establishes the second part and completes the proof. ■

Armed with this lemma, the incentive compatibility constraints in this case are,

$$v(p, y^{m+1} - T) \geq u(y^{m+1} - T + t, \overline{q}), \tag{A7}$$

$$u(y^m - T + t, \overline{q}) \geq v(p, y^m - T). \tag{A8}$$

The Lagrangian expression for this problem can then be written as

$$\begin{aligned}\mathcal{L} &= \sum_{l=1}^{m}\gamma^l u(y^l - T + t, \overline{q}) + \sum_{h=m+1}^{H}\gamma^h v(p, y^h - T) + \mu\left[T - (p\overline{q} + t)\sum_{l=1}^{m}\pi^l\right] \\ &+ \lambda^{m+1}\left[v(p, y^{m+1} - T) - u(y^{m+1} - T + t, \overline{q})\right] + \lambda^m\left[u(y^m - T + t, \overline{q}) - v(p, y^m - T)\right],\end{aligned}$$

with the first-order conditions,

$$\frac{\partial\mathcal{L}}{\partial t} = \sum_l \gamma^l u_c^l - \mu\sum_l \pi^l - \lambda^{m+1}u_c^{m+1} + \lambda^m u_c^m = 0, \tag{A9}$$

$$\frac{\partial\mathcal{L}}{\partial\overline{q}} = \sum_l \gamma^l u_{\overline{q}}^l - p\mu\sum_l \pi^l - \lambda^{m+1}u_{\overline{q}}^{m+1} + \lambda^m u_{\overline{q}}^m = 0, \tag{A10}$$

$$\frac{\partial\mathcal{L}}{\partial T} = -\sum_l \gamma^l u_c^l - \sum_h \gamma^h v_y^h + \mu - \lambda^{m+1}(v_y^{m+1} - u_c^{m+1}) - \lambda^m(u_c^m - v_y^m) = 0. \tag{A11}$$

Now, given that $u_{\overline{q}}^l/u_c^l$ increases with y^l (because c^l increases and $\overline{q}$ remains same), one cannot have $u_{\overline{q}}^l/u_c^l = p$ for all values of l (if $m > 1$). Consequently, one cannot have first-best redistribution in this case. Moreover, it follows from equations (A9)–(A10) that

$$\frac{\sum_l \gamma^l u^l_{\overline{q}} + \lambda^m u^m_{\overline{q}}}{\sum_l \gamma^l u^l_c + \lambda^m u^m_c} = \frac{\mu p \sum_l \pi^l + \lambda^{m+1} u^{m+1}_{\overline{q}}}{\mu \sum_l \pi^l + \lambda^{m+1} u^{m+1}_c}. \tag{A12}$$

This equation tells us that $u^l_{\overline{q}}/u^l_c = p$, if there is one group of poor people (so that $l = 1 = m$) and $\lambda^{m+1} = \lambda^2 = 0$.

Providing many varieties of q: The above discussion alerts us to the possibility of first-best redistribution if one offers as many different bundles of quality and cash as there are poor groups of individuals. Let the government offer q at differentiated quality levels. Specifically, let $\overline{q}_l$ and t_l denote the in-kind and conditional cash transfers to the l-type poor. On the basis of Lemma A1,[18] one can now limit the number of incentive compatibility constraints that has to be taken into account. It will be sufficient to ensure that an individual with income level y^{k+1} does not participate in a cash-cum-in-kind-transfer scheme characterized by $(\overline{q}_k, t_k)$ but that a person with income y^k does (for all $k = 1, 2, \ldots, H-1$.) If these conditions are satisfied, no individuals with incomes greater than y^{k+1} would participate in the $(\overline{q}_k, t_k)$ program. And if the person with y^k chooses $(\overline{q}_k, t_k)$, he will not choose the bundle that is meant for individuals with higher income levels.

To characterize the utility possibility frontier then, one has to determine $\overline{q}_l, t_l$, for $l = 1, 2, \ldots, m$, and T in order to maximize

$$\sum_{l=1}^{m} \gamma^l u(y^l - T + t_l, \overline{q}_l) + \sum_{h=m+1}^{H} \gamma^h v(p, y^h - T),$$

subject to the government's budget constraint,

$$T - \sum_{l=1}^{m} \pi^l (p\overline{q}_l + t_l) \geq 0,$$

---

[18] This lemma was proved for one variety of q. But a similar result holds with many varieties of q as well.

and the incentive compatibility constraints,

$$\begin{aligned}
u(y^l - T + t_l, \overline{q}_l) &\geq u(y^l - T + t_{l-1}, \overline{q}_{l-1}), \qquad l = 2, 3, \ldots, m, \\
v(p, y^{m+1} - T) &\geq u(y^{m+1} - T + t_m, \overline{q}_m), \\
u(y^l - T + t_l, \overline{q}_l) &\geq u(y^l - T + t_{l+1}, \overline{q}_{l+1}), \qquad l = 2, 3, \ldots, m, \\
u(y^m - T + t_m, \overline{q}_m) &\geq v(p, y^m - T).
\end{aligned}$$

Let λ_l denote the downward incentive compatibility constraint for an individual with income l choosing the bundle $(\overline{q}_l, t_l)$ over the bundle $(\overline{q}_{l-1}, t_{l-1})$ for $l = 2, 3, \ldots, m$, with λ^{m+1} corresponding to not participating in the program (and thus enjoying a utility level of $v(p, y^{m+1} - T)$ over the choice of $(\overline{q}_m, t_m)$. Similarly, let δ_l $(l = 1, 2, \ldots, m-1)$ denote the upward incentive compatibility constraint for an individual with income l choosing the bundle $(\overline{q}_l, t_l)$ over the bundle $(\overline{q}_{l+1}, t_{l+1})$, with δ^m corresponding to the choice of $(\overline{q}_m, t_m)$ over not participating (and thus enjoying a utility level of $v(p, y^m - T)$. Denote the utility of a person with income y^k who chooses the bundle $(\overline{q}_j, t_j)$ by $u^{k,j}$.

The Lagrangian expression for this problem is

$$\begin{aligned}
\mathcal{L} &= \sum_{l=1}^{m} \gamma^l u(y^l - T + t_l, \overline{q}_l) + \sum_{h=m+1}^{H} \gamma^h v(p, y^h - T) + \mu \left[T - \sum_{l=1}^{m} \pi^l (p\overline{q}_l + t_l) \right] \\
&+ \sum_{l=2}^{m} \lambda^l \left[u(y^l - T + t_l, \overline{q}_l) - u(y^l - T + t_{l-1}, \overline{q}_{l-1}) \right] + \lambda^{m+1} \left[v(p, y^{m+1} - T) \right. \\
&- \left. u(y^{m+1} - T + t_m, \overline{q}_m) \right] + \sum_{l=1}^{m-1} \delta^l \left[u(y^l - T + t_l, \overline{q}_l) - u(y^l - T + t_{l+1}, \overline{q}_{l+1}) \right] \\
&+ \delta^m \left[u(y^m - T + t_m, \overline{q}_m) - v(p, y^m - T) \right].
\end{aligned}$$

Rearranging the terms, one can rewrite the Lagrangian expression as

$$\begin{aligned}
\mathcal{L} &= (\gamma^1+\delta^1)u(y^1-T+t_1,\overline{q}_1) - \lambda^2 u(y^2-T+t_1,\overline{q}_1) + \mu T - \mu\pi^1(p\overline{q}_1+t_1) \\
&+ \lambda^{m+1} v(p, y^{m+1}-T) - \delta^m v(p, y^m - T) + \sum_{h=m+1}^{H} \gamma^h v(p, y^h - T) \\
&+ \sum_{l=2}^{m} \Big[(\gamma^l+\lambda^l+\delta^l) u(y^l-T+t_l,\overline{q}_l) - \lambda^{l+1} u(y^{l+1}-T+t_l,\overline{q}_l) \\
&- \delta^{l-1} u(y^{l-1}-T+t_l,\overline{q}_l) - \mu\pi^l(p\overline{q}_l+t_l) \,\Big].
\end{aligned}$$

The first-order conditions are, for all $l = 2, 3, \ldots, m$,

$$\begin{aligned}
\frac{\partial\mathcal{L}}{\partial t_1} &= (\gamma^1+\delta^1)u_c^1 - \lambda^2 u_c^{2,1} - \mu\pi^1 = 0, && \text{(A13)} \\
\frac{\partial\mathcal{L}}{\partial t_l} &= (\gamma^l+\lambda^l+\delta^l)u_c^l - \lambda^{l+1} u_c^{l+1,l} - \delta^{l-1} u_c^{l-1,l} - \mu\pi^l = 0, && \text{(A14)} \\
\frac{\partial\mathcal{L}}{\partial \overline{q}_1} &= (\gamma^1+\delta^1)u_{\overline{q}}^1 - \lambda^2 u_{\overline{q}}^{2,1} - \mu p\pi^1 = 0, && \text{(A15)} \\
\frac{\partial\mathcal{L}}{\partial \overline{q}_l} &= (\gamma^l+\lambda^l+\delta^l)u_{\overline{q}}^l - \lambda^{l+1} u_{\overline{q}}^{l+1,l} - \delta^{l-1} u_{\overline{q}}^{l-1,l} - \mu p\pi^l = 0, && \text{(A16)} \\
\frac{\partial\mathcal{L}}{\partial T} &= -(\gamma^1+\delta^1)u_c^1 + \lambda^2 u_c^{2,1} + \mu - \lambda^{m+1} v_y^{m+1} + \delta^m v_y^m - \sum_h \gamma^h v_y^h \\
&- \sum_{l=2}^{m} \Big[(\gamma^l+\lambda^l+\delta^l)u_c^l - \lambda^{l+1} u_c^{l+1,l} - \delta^{l-1} u_c^{l-1,l} \Big] = 0. && \text{(A17)}
\end{aligned}$$

Dividing (A15) by (A13) and (A16) by (A14) yield, for all $l = 2, 3, \ldots, m$,

$$\begin{aligned}
\frac{u_{\overline{q}}^1}{u_c^1} &= \frac{p\mu\pi^1 + \lambda^2 u_{\overline{q}}^{2,1}}{\mu\pi^1 + \lambda^2 u_c^{2,1}} = p + \frac{\lambda^2 u_c^{2,1}(u_{\overline{q}}^{2,1}/u_c^{2,1} - p)}{\mu\pi^1 + \lambda^2 u_c^{2,1}}, \\
\frac{u_{\overline{q}}^l}{u_c^l} &= \frac{p\mu\pi^l + \lambda^{l+1} u_{\overline{q}}^{l+1,l} + \delta^{l-1} u_{\overline{q}}^{l-1,l}}{\mu\pi^l + \lambda^{l+1} u_c^{l+1,l} + \delta^{l-1} u_c^{l-1,l}} \\
&= p + \frac{\lambda^{l+1} u_c^{l+1,l}(u_{\overline{q}}^{l+1,l}/u_c^{l+1,l} - p) + \delta^{l-1} u_c^{l-1,l}(u_{\overline{q}}^{l-1,l}/u_c^{l-1,l} - p)}{\mu\pi^l + \lambda^{l+1} u_c^{l+1,l} + \delta^{l-1} u_c^{l-1,l}}.
\end{aligned}$$

If none of the incentive compatibility constraints are binding, $\lambda^{l+1} = \delta^l = 0$, for all $l = 1, 2, \ldots, m$, and we have a first-best solution.[19] In the second-best solution, one cannot

---

[19] A first-best solution can be constructed directly in an analogous manner to the two-income type

a priori determine the direction of distortion in the consumption of the indivisible good by the various groups of poor people. It will all depend on which incentive compatibility constraints are binding and which are not.

There arises an additional complication here if one wishes also to redistribute between the various rich groups. This seem a natural concern when there are "rich" people of various degrees. Of course, one way of doing this is to designate only one group of individuals as rich, with the aim of redistributing from this group to all other groups. This would require the government to offer $H-1$ different $\overline{q}_l, t_l$ bundles. Leaving this case aside, the possibility of first-best redistribution between the rich groups hinges crucially on the type of information available to the government. If quality levels are publicly observable, one can impose a nonlinear tax conditioned on the purchase of q^h ($h = m+1, m+2, \ldots, H$). However, if this were the case, one could use the same scheme to effect first-best redistribution between the rich and poor as well. Other tax schemes, like linear commodity taxation, which does not rest on public observability of

---

case. Let N_{max} denote the value of $N > 0$ that satisfies the downward incentive compatibility constraint of an individual with income y^{m+1},

$$v\left(p, y^{m+1} - \frac{\sum \pi^l}{\sum \pi^h} N\right) \geq u\left(y^{m+1} + N - pq(p, y^m + N), q(p, y^m + N)\right),$$

as an equality. Then set, for any $0 < N < N_{max}$, and all $l = 1, 2, \ldots, m$,

$$\overline{q}_l = q(p, y^l + N), \qquad t_l = \frac{N}{\sum \pi^h} - p\overline{q}_l, \qquad T = \frac{\sum \pi^l}{\sum \pi^h} N.$$

With all poor groups receiving the same net transfer N, every group will be happiest with his own bundle of $(t_l, \overline{q}_l)$ with $\overline{q}_l$ being the bundle l would buy for himself if he were to receive N in cash. The bundles also satisfy the government's budget constraint, and we have a first-best allocation.

Observe also that other first-best allocations will be made feasible by allowing N to vary for different people. Redefine N_{max} accordingly, and let $0 < N^m < N^{m-1} < \ldots < N^1 < N_{max}$ denote the net transfers to poor groups with incomes $y^m > y^{m-1} > \ldots > y^1$. One needs to set, for $l = 1, 2, \ldots, m$,

$$\overline{q}_l = q(p, y^l + N^l), \qquad t_l = N^l + \frac{\sum \pi^l N^l}{\sum \pi^h} - p\overline{q}_l, \qquad T = \frac{\sum \pi^l N^l}{\sum \pi^h},$$

such that

$$u(y^l - T + t_l, \overline{q}_l) \geq u(y^l - T + t_{l-1}, \overline{q}_{l-1}).$$

Observe also that, given the normality of q, if l does not choose the $(\overline{q}_{l-1}, t_{l-1})$ bundle, he will not choose the bundles for groups $l-2, l-1, \ldots, 1$ either.

consumption levels, coupled with identical lump-sum rebates, can achieve some degree of redistribution between the rich groups but they will be second best.

Proposition A1 summarizes our results in this case.

Proposition A1 *Assume there are many groups of poor and many groups of rich people. Then:*

(i) If only one variant of the indivisible good is provided publicly, the cash-cum-in-kind-transfer scheme is second-best (although it will dominate the Besley and Coate scheme).

(ii) If the indivisible good is provided publicly in as many variants as the designated number of poor groups, with each variant being combined with a different level of conditional cash transfers, the cash-cum-in-kind-transfer scheme consists of first- and second-best solutions.

A3 An Example

Assume there are equal numbers of rich and poor persons ($\pi^h = \pi^l = .5$) who have identical Cobb-Douglas preferences given by

$$u = (cq)^{0.25}.$$

Further, set $p = 1$. This utility function yields the demand functions $c = q = 0.5y$, for any net income level y.

1. *Besley and Coate solution:* It is simple to show that,

$$Q^{min} = \frac{2y^l}{9};\ \ Q^{max} = \frac{2y^h}{9};\ \ \widetilde{Q} = \left(1 - \sqrt{0.5}\right) y^l;\ \ Q^* = \frac{2y^l}{3};\ \ \widehat{Q} = y^l,$$

with

$$u^l(\overline{Q}) = \left[(y^l - 0.5\overline{Q})\overline{Q}\right]^{0.25}. \tag{A18}$$

Observe that $Q^{max} < \widetilde{Q}$ if $y^h < 4.5\left(1 - \sqrt{0.5}\right) y^l$, and $Q^{max} > \widehat{Q}$ if $y^h > 4.5y^l$. We also have $y^h \gtreqless 3y^l \Rightarrow Q^{max} \gtreqless Q^*$.

2. *First-best conditional cash transfers:* We now have,

$$\overline{q}(N) = q(p, y^l{+}N) = 0.5(y^l{+}N);\;\; t(N) = 0.5(3N{-}y^l);\;\; T(N) = N;\;\; N_{FB}^{max} = \frac{(y^h - y^l)^2}{4y^h},$$

with

$$u^l(N) = \Big[0.5(y^l + N)\Big]^{0.5}. \tag{A19}$$

Comparing the maximum attainable utility levels for the poor under the first-best conditional cash transfers (i.e. when $N = N_{FB}^{max} = (y^h - y^l)^2/4y^h$) and under the Besley and Coate scheme (i.e. when $\overline{Q} = Q^{max} = 2y^h/9$), we have

$$u^l(N_{FB}^{max}) - u^l(Q^{max}) = \left[\frac{y^l}{2} + \frac{(y^h - y^l)^2}{8y^h}\right]^{0.5} - \left[\left(y^l - \frac{y^h}{9}\right)\frac{2y^h}{9}\right]^{0.25}.$$

One can easily establish that

$$\begin{array}{lrcl}
\text{Case (i):} & y^h > 3y^l & \Rightarrow & u^l(N_{FB}^{max}) > u^l(Q^{max}), \\
\text{Case (ii):} & y^h = 3y^l & \Rightarrow & u^l(N_{FB}^{max}) = u^l(Q^{max}), \\
\text{Case (iii):} & 1.42002y^l < y^h < 3y^l & \Rightarrow & u^l(N_{FB}^{max}) < u^l(Q^{max}), \\
\text{Case (iv):} & y^l < y^h < 1.42002y^l & \Rightarrow & u^l(N_{FB}^{max}) > u^l(Q^{max}).
\end{array}$$

The first inequality shows that, as demonstrated formally in the text, whenever $Q^{max}(= 2y^h/9) > Q^*(= 2y^l/3)$, one attains a higher maximum utility for the poor under cash transfers. Case (ii) shows that the two solutions are identical when $Q^{max} = Q^*$. The last two inequalities indicate that when $Q^{max} < Q^*$, either policy may result in the maximum attainable utility for the poor.

3. *Second-best conditional cash transfers:* Denote the solutions under the second-best conditional cash transfers by SB. Assume that $y^l = 1$, and generate an example of the above four cases by setting values for y^h that are greater than 3, equal to 3, between 1.42 and 3, and between 1 and 1.42. The *maximum* attainable utility levels for the poor, under Besley and Coate scheme, first-best conditional cash transfers and second-best conditional cash transfers, are then calculated as:

- Case (i), $y^h = 5$:

$$u^l(Q^{max}) = 0.838389, \quad u^l(N_{FB}^{max}) = 0.948683, \quad (u^l)_{SB}^{max} = 1.00576.$$

- Case (ii), $y^h = 3$:

$$u^l(Q^{max}) = 0.816497, \quad u^l(N_{FB}^{max}) = 0.816497, \quad (u^l)_{SB}^{max} = 0.850719.$$

- Case (iii), $y^h = 2$:

$$u^l(Q^{max}) = 0.766776, \quad u^l(N_{FB}^{max}) = 0.750, \quad (u^l)_{SB}^{max} = 0.768101.$$

- Case (iv), $y^h = 1.4$:

$$u^l(Q^{max}) = 0.715932, \quad u^l(N_{FB}^{max}) = 0.717137, \quad (u^l)_{SB}^{max} = 0.723182.$$

It is also interesting to note that when $y^h = 2$ (i.e. in case (iii)), if one sets $\gamma^l = 0.811446$, then $u_{SB}^l = 0.766776$ (as opposed to $u_{SB}^l = 0.768101$ which is attained when $\gamma^l = 1$). This coincides with the Besley and Coate solution where $u^l(Q^{max})$ attains its *maximal* value also at 0.766776.